HIGHER SECONDARY MODEL EXAMINATION, FEBRUARY - 2015

Max. Score : 80

Part III **Time : 2½ Hrs**

Second Year **MATHEMATICS (SCIENCE)** **Cool- off Time : 15 Mts**

ANSWER KEY

1. (i) Consider f: R→R defined by f(x) = 4x + 3

(a) $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow 4x_1 = 4x_2 \Rightarrow x_1 = x_2$. Hence f is one – one.

(b) For every y Є R, there exists [y – 3] / 4 = x Є R such that f (x) = f [[y – 3] / 4] = y .

Therefore, f is invertible

(ii) Let * be a binary operation on N of natural numbers defined by a*b = **L**.C.M of a and b.

(i) * is commutative, since, a*b = **L**.C.M of a and b = **L**.C.M of b and a = b *a

(ii) * is associative, a * (b *c) = **(**a * b) *c

2. Consider a 2 X 2 matrix A = $[a_{ij}]$, where $a_{ij} = |2i - 3j|$

(a)

$$A = \begin{bmatrix} 1 & 4 \\ 1 & 2 \end{bmatrix} \qquad A + A^T = \begin{bmatrix} 2 & 5 \\ 5 & 4 \end{bmatrix}$$

(c) The inverse of A = ½. $\begin{bmatrix} -2 & 4 \\ 1 & -1 \end{bmatrix}$

3. (a) The principal value of $\sin^{-1}(-1/2)$ is **$\underline{-\pi / 6}$**

(b) $$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = \tan^{-1}\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta} = \tan^{-1}\frac{\sec\theta-1}{\tan\theta} = \tan^{-1}\frac{1-\cos\theta}{\sin\theta}$$

$$\tan^{-1}[\tan\theta/2] = \theta/2 = \underline{\tfrac{1}{2}\tan^{-1}x}$$

4. (a) Vertices are (3, 8), (- 4, 2) and (5, 1).

$$\text{Area} = \tfrac{1}{2} . \begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix} = \underline{\tfrac{1}{2} . 61}$$

(i) $$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \mathbf{0}$$

Hence, $A^2 - 5A + 7I = 0$

$A^2 - 5A + 7I = 0 \Rightarrow A^2 A^{-1} - 5AA^{-1} + 7IA^{-1} = 0\,A^{-1}$

$\Rightarrow A - 5I + 7A^{-1} = 0$

$\Rightarrow -7A^{-1} = A - 5I$

$\Rightarrow A^{-1} = -1/7. [A - 5I]$

$$=> A^{-1} = 1/7. \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$X = A^{-1}B = 1/7. \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} . \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$ => **x = 3/7 , y = 19/7**

5. (a) $5k + 1 = 15 - 5 = 10$ => **k = 9/5**

(b) $y = x^{\sin x}$ => $\log y = \sin x \log x$ => $y_1 / y = \sin x . [1/x] + \log x \cos x$

=> $y_1 = x^{\sin x} \{\sin x . [1/x] + \log x \cos x\}$

(c) $y = \sin^{-1} x$ => $y_1 = 1/\sqrt{1 - x^2}$ => $y_1 \sqrt{1 - x^2} = 1$ => $y_2 \sqrt{1 - x^2} + (-2x) y_1 = 0$

$(1 + x^2) y_2 - x y_1 = 0$

6. (a) $S = 2 \pi rh.$

$$\frac{R}{H} = \frac{R - r}{h} \Rightarrow h = (R - r) H / R$$

$S = 2 \pi r [(R - r) H / R] = 2 \pi rH - 2 \pi r r H / R$

(b) $S = 2 \pi rH - 2 \pi r^2 H / R$

$dS / dr = 2 \pi H - 4 \pi r H / R$

$d^2 S / dr^2 = - 4 \pi H / R$

$dS / dr = 2 \pi H - 4 \pi r H / R = 0 \Rightarrow r = R/2$

$d^2 S / dr^2 (r = R/2) = - 4 \pi H / R < 0$

Hence, S is maximum, when **r = R/2**

7. (a) $\int \tan x \, dx = \log \sec x + C$

(b) $\int \cos^2 (2x + 3) dx = ½ . \int [1 + \cos 2 (2x + 3) /2]dx = ½ . \{[x + \sin (4x + 6)]/ 4\} + C$

(c) $$\frac{2x}{x^2 + 3x + 2} = \frac{A}{(x + 1)} + \frac{B}{(x + 2)} \Rightarrow 2x = A (x + 2) + B (x + 1)$$ **[2]**

=> $-2 = A (-1 + 2)$ => **A = -2**

=> $-4 = B (-2 + 1)$ => **B = 4**

(c) $$\int \frac{2x}{x^2 + 3x + 2} dx = \int \frac{-2}{(x + 1)} dx + \int \frac{4}{(x + 2)} dx \Rightarrow -2 \log (x + 1) + 4 \log (x + 2) + C$$

8. (a) $\int_0^1 \log\left(\frac{x}{x-1}\right) dx = \int_0^1 \log\left(\frac{1-x}{-x}\right) dx = \int_0^1 \log\left(\frac{x-1}{x}\right) dx$

$2I = \int_0^1 \log 1\, dx = \mathbf{0} \Rightarrow I = 0$

(b) $\int_{-1}^{2} |x^3 - x|\, dx = \int_{-1}^{0} (x^3 - x)\, dx + \int_0^1 -(x^3 - x)\, dx + \int_1^2 (x^3 - x)\, dx = 11/2$

9. (a) $\int_1^4 y\, dx = \int_1^4 \sqrt{x}\, dx = 14/3$

(b) Eqn. to AB $y = 2(x - 1)$, Eqn. to BC $y - 2 = -(x - 2)$, Eqn. to AC $y = ½.(x - 1)$

Eqn. to BC $y = 4 - x$

Area = $\int_1^2 2(x-1)\, dx + \int_2^3 (4 - x)\, dx - \int_1^3 ½.(x - 1)\, dx$ = **<u>3 / 2</u>**

10. (a) Order = 3 and degree = 2

(b) The general solution of the differential equation $y' = e^{x-y}$ is **<u>$e^y - e^x = C$</u>**

(c)

$$y' = \frac{2xy}{1 + x^2} + x^2 + 2$$

(i) Integrating Factor = $1/ 1 + x^2$

(ii) $y = (1 + x^2)(x + \tan^{-1} x) + C$

11. Consider the points A(2, 3, 4), B(4, 3, 2) and C (5, 2, -1).

(a) Projection of $\overline{BC}$ on $\overline{AB}$ = **<u>$4 / \sqrt{2}$</u>**

BC = i – j – 3k , AB – 2i + 0j – 2k, BC. AB = 2 + 0 + 6 = **<u>8</u>** , $|AB| = \sqrt{4 + 4}$ = **<u>$2\sqrt{2}$</u>**

(b) Area of the triangle ABC = ½. $|BC \times AB|$ \

BC X AB = 2i – 4j + 2k

$|BC \times AB| = \sqrt{4 + 16 + 4}$ = **<u>$\sqrt{24}$</u>**

Area = $\sqrt{24}/2$ = <u>$\sqrt{6}$</u>

12. $\bar{\mathbf{a}}$ = i + j + k, and $\bar{\mathbf{b}}$ = i + 2j + 3k **$\bar{a} + \bar{b}$ = 2i + 3j + 4k , $\bar{a} - \bar{b}$ = -j - 2k**

(a) Unit vector perpendicular to each of **a** + **b** and **a** – **b** is **<u>- 2i – 4j - 2k /$\sqrt{24}$</u>**

a . b = 1 + 2+ 3 = 6 $|\bar{a}|$ = **<u>$\sqrt{3}$</u>** , $|\bar{b}|$ = **<u>$\sqrt{14}$</u>**

(b) The angle between **a** and **b** , $\cos\theta$ = **<u>$6 / \sqrt{3}\,.\sqrt{14}$</u>**

13. (a) The equation of the plane through the points (3, -1, 2), (5, 2, 4) and (1, 1, -6) is

$\underline{10x - 6y - z - 34 = 0}$

(b) The perpendicular distance from the point (6, 5, 9) to **$\underline{10x - 6y - z - 34 = 0}$** is **$\underline{13 / \sqrt{137}}$**

14. Consider the lines

$r = 2i + j + \lambda(i - 2j + k)$ $r = 2i + j - k + \mu(3i - 5j + 3k)$

(a) $\cos\theta$ = **$\underline{16 / \sqrt{6} \, . \sqrt{43}}$**

(b) $(a_2 - a_1) = -k$, $(b_1 \times b_2) = -i + k$, $(a_2 - a_1) . (b_1 \times b_2) = -1$

Distance d = **$\underline{1/\sqrt{2}}$**

15. (a) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A) - P(B) + P(A) . P(B) = [1 - P(A)] [1 - P(B)] = P(A') . P(B')$

Hence A' and B' are also independent.

(b) **A :** the man reports that 6 occurs

E_1 : six occurs

E_2 : six does not occur

$P(E_1) = 1/6$, $P(E_2) = 5/6$, $P(A/E_1) = ¾$, $P(A/E_2) = ¼$

$\underline{P(E_1/A) = 3/8}$

OR

15. (a) The probability of occurrence of at least one of A and B is = P(AUB)

$= P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A) . P(B)$

$= P(A) + P(B) P(A)'$

$= 1 - P(A') + P(B) P(A')$

$= 1 - P(A') .[1 - P(B)]$

$= 1 - P(A') P(B')$

(b) A die is thrown 6 times " getting an odd number " is a **success**

n = 6, p = 3/6 = ½ , q = ½

(i) **P(**5 success) = **3/32**

(ii) **P(**at least 5 success) = 7/64

(ii) **P(**at most 5 success) = 63 / 64

16.

corner points	**corresponding value of Z**	
(0, 10)	**90**	
(5, 5)	**60**	← **minimum value**
(15, 15)	**180**	← **maximum value**
(0, 20)	**180**	← **maximum value**

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