HIGHER SECONDARY MODEL EXAMINATION, FEBRUARY 2015

Max. Score: 80

Part III **Time : $2\frac{1}{2}$ Hrs.**

First Year **MATHEMATICS (COMMERCE)** **Cool off Time: 15 Mts.**

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ANSWER KEY

[1] U = $\{1, 2, 3, 4, 5, 6, 7\}$, A = $\{1, 2, 3, 4, 7\}$, B = $\{2, 3, 5, 7\}$,

AUB = $\{1, 2, 3, 4, 5, 7\}$, 2^6 elements, $A' = \{5, 6\}$,$B' = \{1, 4, 6\}$,$A' \cap B' = \{6\}$,

[2] $f(x) = \frac{x^2+2x+3}{x^2-8x+12} = \frac{x^2+2x+3}{(x-2)(x-6)}$, Domain = $R - \{2, 6\}$, 10 x = 9 , x = 9 / 10,

Draw the graph of f(x) = $|x|$

[3] x = $\frac{\pi}{4}$, sin (45 – 30) = sin 45 cos 30 – cos 45 sin 30 = $\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x} = \frac{2\cos 3x\cos x+\cos 2x}{2\sin 3x\cos x+\sin 2x} = \frac{\cos 2x\,(2\cos x+1)}{\sin 2x\,(2\cos x+1)}$ = cot 3x

[4] P (n) : 1 + 3 + 3^2 + ……..+ $3^{n-1} = \frac{3^n-1}{2}$

P (1) : 1 = $\frac{3^1-1}{2}$ => 1 = 1 => P(1) is true

Assume that P (k) : 1 + 3 + 3^2 + ……..+ $3^{k-1} = \frac{3^k-1}{2}$ is true

1 + 3 + 3^2 + ……..+ $3^{n-1} + 3^n = \frac{3^n-1}{2} + 3^n = \frac{3^n[2+1]-1}{2} = \frac{3^n.3-1}{2} = \frac{3^{n+1}-1}{2}$ = P(n+1)

[5] $i^{-35} = (i^4)^{-8}.i^{-3} = i^{-3} = i$

z = 1 + i $\sqrt{3}$, r = 2 , $\cos\theta = \frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$, $\theta = \frac{\pi}{3}$, z = 1 + i $\sqrt{3}$ = 2 (cos $\frac{\pi}{3}$ + i sin $\frac{\pi}{3}$)

x^2 + x+ 1 = 0 => x = $\frac{-1\pm\sqrt{1-4}}{2} = \frac{-1\pm i\sqrt{3}}{2}$

[6] x < 4

Solve x + 2y ≤ 8, 2x + y ≤ 8

[7] n + 1 , $(-1)^r 6C_r (x^2)^{6-r} y^r$, $9C_r (x)^{9-r} (2y)^3 = x^6 y^3$ => r = 3. **<u>Ans. 672</u>**

[8] $n!$,

n P_5 = 42 n P_3 => $n(n-1)(n-2)(n-3)(n-4) = 42.\,n(n-1)(n-2)$

$\Rightarrow (n-3)(n-4) = 42. \Rightarrow (n+3)(n-10) = 0$. n = - 3 , **n = 10**

9 x 8 x 7 x 6 = **3024**

OR

[8] n = 10 , 21 C_2 = 210 , 15 C_{11} = 32760 , 5 C_2. 6 C_3. 4 C_2 = 10. 20. 6 = **1200**

[9] 9, $\frac{1499}{2}$ [3 + 2999] = **2249999** , a = 3 , d = 3 , a_6 = 18, **Numbers are 6, 9, 12, 15**

OR

[9] 4, $\frac{5}{9}\frac{(10^n-1)}{9}$, $S_n = \sum n(n+1) = \sum n^2 + \sum n = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

[10] slope, m = $\frac{4--2}{-1-3} = \frac{-3}{2}$, distance, d = $|\frac{3x3-4x5-26}{\sqrt{3^2+4^2}}| = \frac{3}{5}$,

$\sqrt{1^2 + (\sqrt{3})^3} = 2$, x $\frac{1}{2}$ + y $\frac{\sqrt{3}}{2} = \frac{8}{2}$, p = 4, $\omega = \frac{\pi}{3}$

[11] 0 , |PQ| = $\sqrt{14}$ = **1. $\sqrt{14}$** , |QR| = $\sqrt{56}$ = **2. $\sqrt{14}$** , |PR| = $\sqrt{126}$ = **3. $\sqrt{14}$** , PQ + QR = PR

[12] a = 6 , y^2 = 4. 6. x => y^2 = 24x. ,

$4x^2 + 9y^2 = 36 \Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1 \Rightarrow$ a = 3, b = 2 , c = $\sqrt{9-4} = \sqrt{5}$, e = $\frac{c}{a} = \frac{\sqrt{5}}{3}$

Latus rectum = 2 $\frac{b^2}{a} = \frac{8}{3}$

[13] 1 , $\lim_{x\to\pi} \frac{\sin(\pi-x)}{\pi(\pi-x)} = \frac{1}{\pi}\lim_{(\pi-x)\to 0} \frac{\sin(\pi-x)}{(\pi-x)} = \frac{1}{\pi} . 1 = \frac{1}{\pi}$

[14] $\frac{d}{dx}\left(\frac{x+\cos x}{\sin x}\right) = \frac{\sin x \frac{d}{dx}(x+\cos x) - (x+\cos x)\frac{d}{dx}(\sin x)}{(\sin x)^2}$

$$= \frac{\sin x\ (1-\sin x) - (x+\cos x)(\cos x)}{(\sin x)^2}$$

$$= \frac{\sin x - x\ cos x - 1}{(\sin x)^2}$$

OR

[14] f'(x) = $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = = \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h} = \lim_{h\to 0} \frac{2\sin(x+\frac{h}{2})COSX}{h}$

$= \lim_{\frac{h}{2}\to 0} \frac{\sin(x+\frac{h}{2})}{\frac{h}{2}} \lim_{h\to 0} \cos x$ = 1. cos x = cos x

[15] It is not true that a good teacher is always a student.
Assume the contrary.

$\sqrt{2}$ is rational

$\sqrt{2} = \frac{p}{q}$, where (p, q) = 1.

p = $\sqrt{2}$ q => $p^2 = 2q^2$

=> $2 \mid p^2$

=> $2 \mid p$

=> $p = 2c$, *where c is an integer.*

=> $p^2 = 4c^2$

=> $2q^2 = 4c^2$

=> $q^2 = 2c^2$

=> $2 \mid q^2$

=> $2 \mid q$

=> $2 \mid p$ and $2 \mid q$ => (p, q) = 2 – contradiction.

Therefore, $\sqrt{2}$ is irrational.

[16] 0.2 , $\{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$, $\frac{6}{36} = \frac{1}{6}$

[17] Mean, = 107 , σ^2 = 2276 , σ = 47.71

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